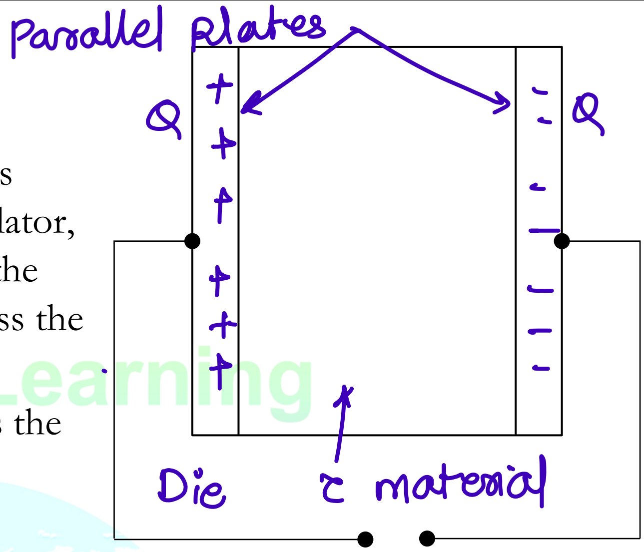
Capacitance



Capacitor and Capacitance:

A capacitor is an electrical component that stores charge on two separated metallic plates. An insulator, sometimes called a dielectric, is placed between the plates to prevent the charge from travelling across the gap. The capacitance, C, is defined as the charge stored, Q, per unit potential difference, V, across the two plates. Therefore, we can write

$$C = \frac{Q}{V} \quad Q \rightarrow (C) \quad V \rightarrow (volt)$$

$1\,\mu F = 10^{-6} F$

$1\,nF = 10^{-9} F$

$1\,pF = 10^{-12} F.$

where capacitance is measured in Farads, F (C/V)

- ✓ When a capacitor is connected to a DC power supply,

e.g. a cell or battery, there is a brief current as the power supply draws electrons from one plate and deposits them on the other plate.

- ✓ This leaves the first plate with charge $+Q$ and the second with charge $-Q$.
- ✓ These charges will be equal and opposite due to the conservation of charge. Current will flow in the circuit until the potential difference between the plates is equal to that of the electromotive force or e.m.f. of the power supply.
- ✓ The dielectric has another purpose: to increase the capacitance of the device by polarizing in the electric field and effectively increasing the charge stored on the plates. Dielectrics have an associated electrical permittivity which describes its ability to polarize and strengthen the charge storage capability of the device. This is why in reality the insulator is rarely a vacuum or just air as these materials do not polarize well (or at all in the case of the vacuum) and so are poor dielectrics

Applications of Capacitors

1. Capacitors are used **to store and discharge large quantities of energy** in a short time period. This makes them useful for short pulses of energy such as camera flashes and touch screens where a short finger press leads to a large buildup of energy in a capacitor.

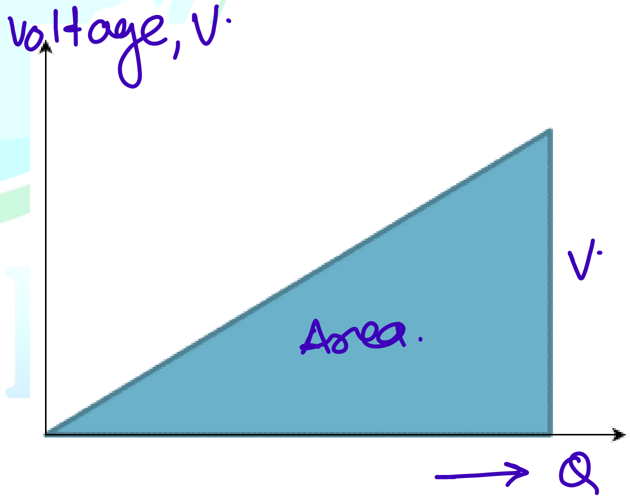
2. They are also integral to **uninterrupted power supplies or UPSs** which are used as backup power supplies when the mains electricity supply fails. UPSs are commonly found in data centers **to protect the hardware** and in **hospitals to maintain a constant power supply to life support machines.**

3. Finally, capacitors are used in the process **of converting alternating current (AC) into direct current (DC)**. Once a sinusoidal AC signal has passed through a full wave rectifier, the current flows in one direction but varies as shown. The current can then be passed through a smoothing circuit in which a capacitor stores energy as the p.d. rises and discharges as it falls.

Energy Stored/Work Done in a Capacitor

Work must be done by the power supply to deposit negatively charged electrons onto the negative plate as like charges repel according to Coulomb's law. Equally, work is done to remove electrons from the positive plate as negative charges are attracted to positive regions.

The graph shows the charge stored on a capacitor plate against the potential difference over the device. As voltage is defined as the electrical potential energy per unit charge the area under the graph must therefore represent the work done in charging up the capacitor and so the energy stored in the capacitor.



Therefore,

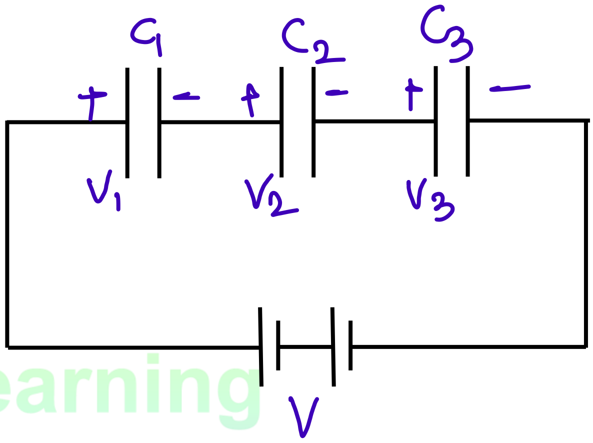
$$\text{Work} = \frac{1}{2}QV$$

but $Q = CV$

$$W = \frac{1}{2}V^2C \quad , \quad W = \frac{Q^2}{2C}$$

Capacitors in Series

- Potential difference(V) across capacitor is different
- Charges (Q) are same



$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C}, \quad V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

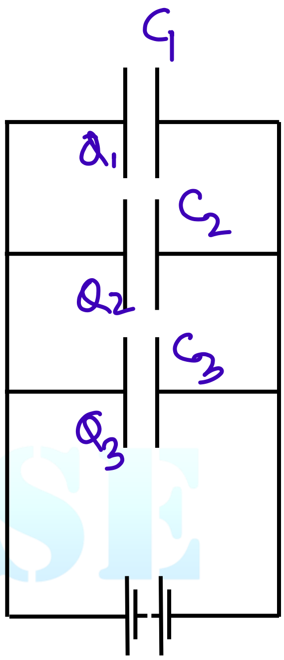
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{Q}{C} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in Parallel

- Potential difference(V) across capacitor is same
- Charges(Q) are different across capacitor



$$Q = Q_1 + Q_2 + Q_3$$

$$Q = CV, \quad Q_1 = C_1V, \quad Q_2 = C_2V, \quad Q_3 = C_3V$$

$$CV = C_1V + C_2V + C_3V$$

$$CV = V(C_1 + C_2 + C_3)$$

$$\boxed{C = C_1 + C_2 + C_3}$$

Charging and discharging of capacitor

Exponential decay- relation between x and y axis

$$x = x_0 e^{-ky}$$

Current

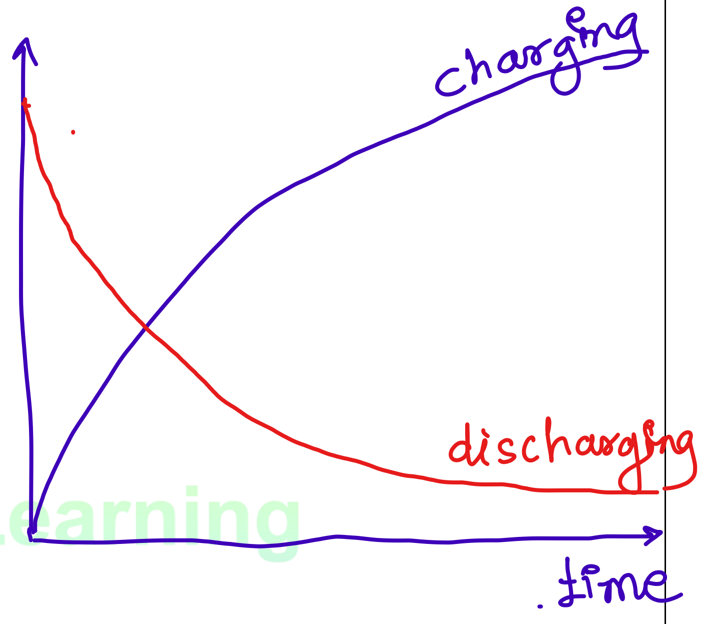
$$I = I_0 e^{\left(-\frac{t}{RC}\right)}$$

Voltage

$$V = V_0 e^{\left(-\frac{t}{CR}\right)}$$

Charges

$$Q = Q_0 e^{\left(-\frac{t}{CR}\right)}$$



Capacitance and Relative Permittivity

$$C \propto A, \quad C \propto \frac{1}{d}$$

$$C = \epsilon_0 \frac{A}{d}$$

ϵ_0 - proportionality constant.

$$\boxed{C = \epsilon_0 \epsilon_r \frac{A}{d}}$$

ϵ_r = relative permittivity